322351(14)

B. E. (Third Semester) Examination, April-May 2021

(New Scheme)

(CSE Engg. Branch)

MATHEMATICS-III

Time Allowed: Three hours

Maximum Marks: 80

Minimum Pass Marks: 28

Note: Part (a) of each question is compulsory.

Attempt any two part from (b), (c) and (d) of each question.

1. (a) Explain Dirichlet's conditions for a fourier expansion of function.

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(b) Find a series of sines and cosines of multiples of x which will represent the function $f(x) = x + x^2$ in the interval $-\pi < x < \pi$. Hence show that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$$

- (c) Find the fourier series for the function f(x), if $f(x) = |\cos x| \text{ in } -\pi < x < \pi.$
- (d) Find out the constant term and the coefficient of the first sin c and cosin c terms in the fourier series of y as given in the following table:

2. (a)
$$L^{-1}(\sqrt{t}) = \dots$$

(b) (i)
$$\int_0^\infty t \, e^{-2t} \cos t \, dt = \frac{3}{25}$$

(ii) Prove that

$$L^{-1}\left\{\frac{4S+5}{(S-1)^2(S+2)}\right\} = 3te^t + \frac{e^t}{3} - \frac{e^{-2t}}{3}$$
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(c) Apply Convolution theorem to evaluate:

$$L^{-1}\left[\frac{S^2}{\left(S^2+a^2\right)\left(S^2+b^2\right)}\right]$$

(d) Solve

$$\frac{d^2x}{dt^2} + 8x = \cos 2t,$$

if
$$x(0) = 1$$
, $x(\pi/2) = -1$.

- 3. (a) State Cauchy's integral formula.
 - (b) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though C. R. equation are satisfied thereof.

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(c) Evaluate

$$\oint_{c} \frac{e^{z}}{\left(z^{2} + \pi^{2}\right)^{2}} dz$$

where C is |z| = 4

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(d) Show that

$$\int_0^{2\pi} \frac{\cos 2\theta \ d\theta}{1 - 2a\cos\theta + a^2} = \frac{2\pi \ a^2}{1 - a^2} \left(a^2 < 1\right)$$

4. (a) From the partial differential equation from

$$z = y^2 + 2t \left(\frac{1}{x} + \log y\right)$$

(b) Solve

$$(x^2 - yz)p + (y^2 - zx)9 = z^2 - xy$$

(c) Solve:

$$r - 4x + 4t = e^{2x + y}$$

(d) Using the method of separation of variables, solve

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$$
, where $u(x,0) = 6 e^{-3x}$

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- 5. (a) Define moment generating function of discrete and continuous probability distribution.
 - (b) In sampling a large number of parts manufactured by a machine, the mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain at least 3 defective parts.
 - (c) Fit a Poission distribution to the set of observations:

$$x : 0 \quad 1 \quad 2 \quad 3 \quad 4$$
 $f : 122 \quad 60 \quad 15 \quad 2 \quad 1 \quad 7$

(d) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and SD of the distribution.

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